Chapter 9
Mechanical Properties of Solids

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1 Marks Questions

1.The stretching of a coil spring is determined by its shear modulus. Why?

Ans . When a coil spring is stretched, neither its length nor its volume changes, there is only the change in its shape. Therefore, stretching of coil spring is determined by shear modulus.

2.The spherical ball contracts in volume by 0.1% when subjected to a uniform normal pressure of 100 atmosphere calculate the bulk modulus of material of ball?

Ans . Volumetric strain = $\frac{\Delta V}{V} = 0.1\% = \frac{0.1}{100} = 10^{-3}$

Normal Stress = 100 atmosphere = 100 X 10^5 = 10^7 N|m^2

∴ Bulk Modulus of the material of the ball is :→

$$K = \frac{Normal\ Stress}{Volumetric\ Strain} = \frac{10^7}{10^{-3}} = 10^{10}\ N\,|\,m^2$$

3.State Hooke's law?

Ans. Hooke's law states that the extension produced in the wire is directly proportional to the load applied within the elastic limit i.e. Acc to Hooke's low,

Stress α Strain

Stress = E x Strain

E = Modulus of elasticity

4.What are ductile and brittle materials?

Ans. Ductile materials are those materials which show large plastic range beyond elastic limit. eg:- copper, Iron

Brittle materials are those materials which show very small plastic range beyond elastic limit. eg:- Cast Iron, Glass.

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5. A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N m^{-2}, what is the maximum load the cable can support?

Ans. Radius of the steel cable, r = 1.5 cm = 0.015 m

Maximum allowable stress = 10^8 N m^{-2}

Maximum stress = $\frac{Maximum\ force}{Area\ of\ cross-section}$

$\therefore$Maximum force = Maximum stress $\times$ Area of cross-section

= $108\times\pi(0.015)^2$

= $7.065\times10^4\ N$

Hence, the cable can support the maximum load of 7 $7.065\times10^4\ N$.

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6. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

Ans. Hydraulic pressure exerted on the glass slab, p = 10 atm = $10\times1.013\times10^5\ Pa$

Bulk modulus of glass, B = $37\times10^9\ Nm^{-2}$

Bulk modulus, $B=\frac{p}{\frac{\Delta V}{V}}$

Where, $\frac{\Delta V}{V}$ = Fractional change in volume

$$\therefore \frac{\Delta V}{V} = \frac{p}{B}$$

$$= \frac{10 \times 1.01310^{5}}{37 \times 10^{9}}$$

$$= 2.73 \times 10^{-5}$$

Hence, the fractional change in the volume of the glass slab is 2.73×10^{-5}.

2 Marks Questions

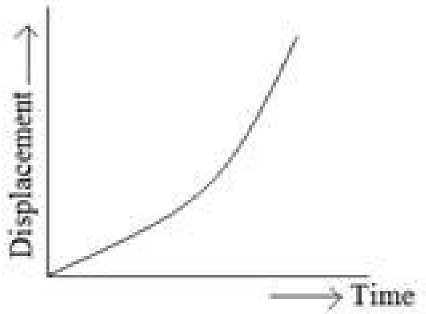
1.Write the characteristics of displacement?

Ans: (1) It is a vector quantity having both magnitude and direction.

(2) Displacement of a given body can be positive, negative or zero.

2.Draw displacement time graph for uniformly accelerated motion. What is its shape?

Ans: The graph is parabolic in shape



3.Sameer went on his bike from Delhi to Gurgaon at a speed of 60km/hr and came back at a speed of 40km/hr. what is his average speed for entire journey.

Ans:

$$v_{av} = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times 60\times 40}{60+40} = 48 km/hr.$$

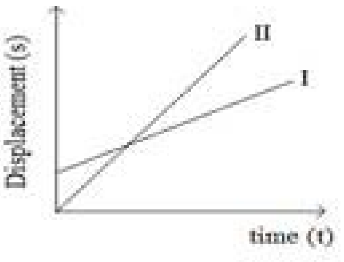
4.What causes variation in velocity of a particle?

Ans: Velocity of a particle changes

(1) If magnitude of velocity changes

(2) If direction of motion changes.

5.Figure. Shows displacement – time curves I and II. What conclusions do you draw from these graphs?



Ans: (1) Both the curves are representing uniform linear motion.

(2) Uniform velocity of II is more than the velocity of I because slope of curve (II) is greater.

6.Displacement of a particle is given by the expression x = $3t^2$ + 7t – 9, where x is in meter and t is in seconds. What is acceleration?

Ans: $x = 3t^2 + 7t - 9$

$$\upsilon = \frac{dx}{dt} = 6t + 7 \text{ m/s}$$

$$a = \frac{d\vartheta}{dt} = 6m/s^2$$

7.A particle is thrown upwards. It attains a height (h) after 5 seconds and again after 9s comes back. What is the speed of the particle at a height h?

Ans: $s = ut + \frac{1}{2}at^2$

As the particle comes to the same point as 9s where it was at 5s. The net displacement at 4s is zero.

$$0 = v \times 4 - \frac{1}{2}(g) \times (4)^2$$

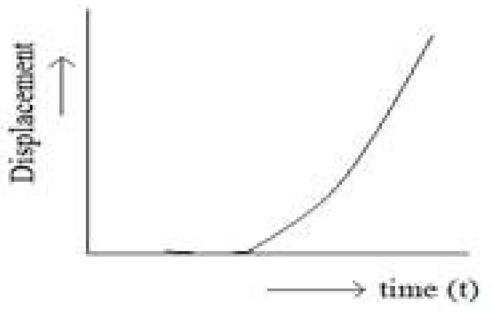
$$4v = \frac{1}{2} \times 9.8 \times 16$$

$$v = 2 \times 9.8$$

$$v = 19.6 m/s$$

8.Draw displacement time graph for a uniformly accelerated motion? What is its shape?

Ans: Graph is parabolic in shape



9.The displacement x of a particle moving in one dimension under the action of constant force is related to the time by the equation where x is in meters and t is in seconds. Find the velocity of the particle at (1) t = 3s (2) t = 6s.

Ans: $t = \sqrt{x} - 3$

$$\sqrt{x} = t + 3$$

$$x = (t+3)^2$$

(i) $v = \frac{dx}{dt} = 2(t+3)$

For $t = 3$ sec $v = 2(3+3) = 12 m/s$

(ii) For $t = 6$ sec $\upsilon = 2(6+3) = 18m/s$

10.A balloon is ascending at the rate of 4.9m/s. A pocket is dropped from the balloon when situated at a height of 245m. How long does it take the packet to reach the ground? What is its final velocity?

Ans: $u = 4.9m/s\ (upward)$

$h = 245m$

For packet (care of free fall) a = g = 9.8m/s^2 (downwards)

$$s = ut + \frac{1}{2}at^2$$

$$245 = -4.9 \times t + \frac{1}{2}(9.8) \times t^2$$

$$4.9t^2 - 4.9t = 245$$

$t = 7.6s\ or\ -5.6s$ Since time cannot be negative

$\therefore$ t = 7.6s

Now $\upsilon = u + at$

$$\upsilon = -4.9 + (9.8)(76)$$

$$\upsilon = 69.6m/s$$

11.A car moving on a straight highway with speed of 126km/hr. is brought to stop within a distance of 200m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop?

Ans: $u = 126km/hr = 35m/s$

$\upsilon = 0\ s = 200m$

$$\upsilon^2 - v^2 = 29s$$

$$a = \frac{\upsilon^2 - v^2}{2s}$$

$$a = \frac{(0)^2 - (126)^2}{2 \times 200} = \frac{(0)^2 - (35)^2}{2 \times 200}$$

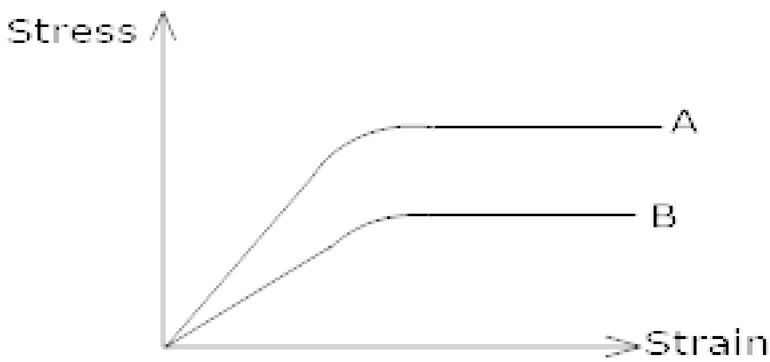
a = -3.06m/s^2 (Retardation)

Now V = u + at

$$t = \frac{V - v}{a} = \frac{0 - 35}{-3.06}$$

t = 11.4s

12.In the following stress – strain curve, which has:-



1) Greater young's Modulus   2) More Ductility 3) More Tensile strength.

Ans .

1) Since young's Modulus is given by the slope of stress – strain graph, Since slop of A is more than that of B, hence it has greater young's Modulus.

2) Ductility is the extent of plastic deformation and it is greater for A.

3) Tensile strength is the direct measure of stress required, from by graph, it is greater for A.

13.A cube is subject to a pressure of 5X10^5 N|m^2. Each side of cube is shortened by 1% find: - 1) the volumetric strain 2) the bulk modulus of elasticity of cube.

Ans . Let l = Initial length of cube.

Initial volume, V = l^3.

Change in length = 0.01% of l = $\frac{1}{100}l$

Final length of each side of cube = $l - \frac{l}{100} = \frac{99}{100}l$

Final volume = $\left(\frac{99l}{100}\right)^3$

Change in Volume, $\Delta V = \left(\frac{99l}{100}\right)^3 - l^3 = l^3\left[\left(\frac{99}{100}\right)^3 - 1\right]$

1) Volumetric Strain; $\frac{\Delta V}{V} = \frac{l^3\left[\left(\frac{99}{100}\right)^3 - 1\right]}{l^3} = \frac{-3}{100} = 0.03$

2) Bulk Modulus, K = $\frac{Normal\ Stress}{Volumetric\ Strain} = \frac{5X10^5}{0.03} = 1.67X10^7 N|m^2$

14.If the potential energy is minimum at r = r$_o$ = 0.74A^O, is the force attractive or repulsive at r = 0.5A^O; 1.9A^O and α?

Ans . Since, potential energy is minimum at r$_O$ = 0.74A^O. therefore interatomic force between two atoms is zero for r$_O$ = 0.74A^O

1) At r = 0.5 A^O (Which is less than r$_o$), the force is repulsive.

2) At r = 1.9A^{O} (Which is greater than r_o), the force is attractive.

3) At r = α, the force is zero.

15.A hollow shaft is found to be stronger than a solid shaft made of same equal material? Why?

Ans.A hollow shaft is found to be stronger than a solid shaft made of equal material because the torque required to produce a given twist in hollow cylinder is greater than that required to produce in solid cylinder of same length and material through same angle.

16.Calculate the work done when a wire of length l and area of cross – section A is made of material of young's Modulus Y is stretched by an amount x?

Ans. Young's Modulus = $\frac{\text{Normal Stress}}{\text{Longitudinal Strain}}$

Y = $\frac{F/A}{l/L}$

F = Force

A = Area

l= change in length

L = original Length

x = change in Length (Given)

Average extension = $\frac{o+x}{2}=\frac{x}{2}$

Now, Work Done = Force. Average extension

$$\Rightarrow Now, y = \frac{FL}{Al}$$

$$F = \frac{YAl}{L}$$

$$\text{work Done} = \frac{YAl}{L} \cdot \frac{x}{2}$$

$$= \frac{YAx}{L} \cdot \frac{x}{2} \quad \left(l = x(given)\right)$$

$$\text{Work Done} = \frac{YAx^2}{2L}$$

17.Water is more elastic than air. Why?

Ans . Since volume elasticity is the reciprocal of compressibility and since air is more compressible than water hence water in more elastic than air.

18.The length of a metal is l_1, when the tension in it is T_1 and is l_2 when tension is l_2. Find the original length of wire?

Ans . Let l = original length of material – wire.

A = original length of metal – wire.

Change in length in the first case = (l_1-l)

Change in length is second case = (l_2-l)

Now, Young Modulus

$$= \frac{\text{Normal Stress}}{\text{Longitudinal Strain}}$$

$$Y = \frac{T/A}{\Delta l/l}$$

Y = Young's Modulus

T = Tension

A = Area

Δl = Change in length

l = Original Length

$$\therefore Y = \frac{T_1}{A} X \frac{l}{(l_1 - l)} \text{ for first case.}$$

$$Y = \frac{T_2}{A} X \frac{l}{(l_2 - l)} \text{ for second case.}$$

Since Young's Modulus remains the same,

So,

$$\frac{T_1}{A} X \frac{l}{(l_1 - l)} = \frac{T_2}{A} X \frac{l}{(l_2 - l)}$$

$$T_1 (l_2 - l) = T_2 (l_1 - l)$$

$$T_1 l_2 - T_1 l = T_2 l1 - l\ T_2$$

$$l (T_2 - T_1) = T_2 l_1 - T_1 l_2$$

$$l = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$$

19.An elastic wire is cut to half its original length. How would it affect the maximum load that the wire can support?

Ans. Since Breaking load = Breaking Stress x Area; so if cable is cut to half of its original length, there is no change in its area hence there is no effect on the maximum load that the wire can support.

20.Define modulus of elasticity and write its various types

Ans. Modulus of elasticity is defined as ratio of the stress to the corresponding strain produced, within the elastic limit.

E (Modulus of elasticity) = $\frac{Stress}{Strain}$

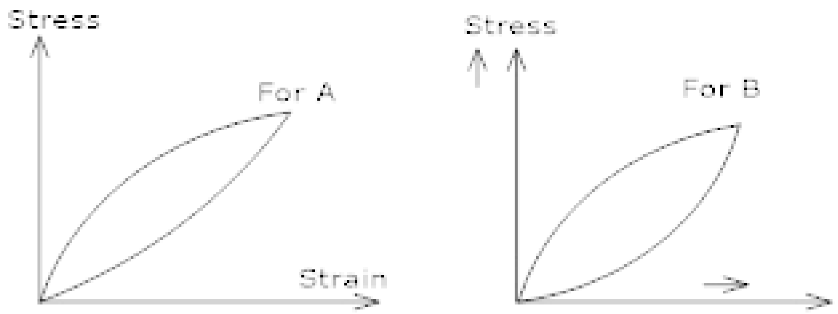
Types of Modulus of elasticity:-

1) Young's Modulus = $\frac{Normal\ Stress}{Longitudinal\ Strain}$

2) Bulk Modulus = $\frac{Normal\ Stress}{Volunetric\ Strain}$

3) Modulus of Rigidity = $\frac{Tangential\ Stress}{Shearing\ Strain}$

21.Two different types of rubber are found to have the stress – strain curves as shown in the figure stress



i) In what ways do these curves suffer from the stress- strain curve of a metal wire?

2) Which of the two rubbers A and B would you prefer to be installed in the working of a heavy machinery

3) Which of these two rubbers would you choose for a car tyre?

Ans.1) Since for the above curves, Hooke's law is not obeyed as the curve is not a straight line Hence such type of curve are called as elastic hysteresis as the materials do not retrace curve during unloading.

2) Rubber B is preferred because area of loop B is more than that of A which shows more absorption power for vibrations which is useful in machinery.

3) Since hysteresis loop is a direct measure of heat dissipation, hence rubber A is preferred over B so to minimize the heating in the car tyres.

22.Which is more elastic rubber or steel? Explain.

Ans. Let length and area of rubber and stead rod = l and a respectively

Let Yr = Young's modulus of elasticity for rubber

Ys = Young's modulus of elasticity for steel when Stretching force F is applied, Let

$\Delta lr =$ Extension in rubber

Δ ls = Extension in steel

Now, Δlr will be greater that Δls.

Now Y = $\dfrac{Fl}{a\Delta l} = \dfrac{Normal\ stress}{Longitudinal\ strain}$.

So, $Y_r = \dfrac{Fl}{a\Delta lr}$; $Ys = \dfrac{Fl}{a\Delta ls}$

Since Δlr >Δls

So, Y r < Ys

Hence more the modulus of elasticity more elastic is the material, so, steel is more elastic than rubber.

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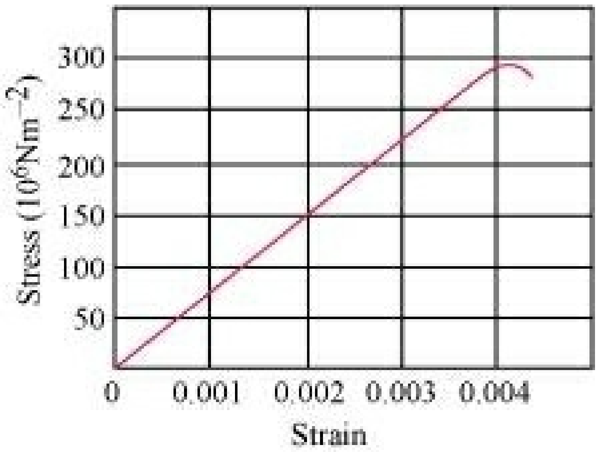
23. Figure 9.11 shows the strain-stress curve for a given material. What are (a) Young's

modulus and (b) approximate yield strength for this material?



Ans. (a) It is clear from the given graph that for stress 150 × 106 N/m2, strain is 0.002.

∴Young's modulus, $Y = \frac{Stress}{Strain}$

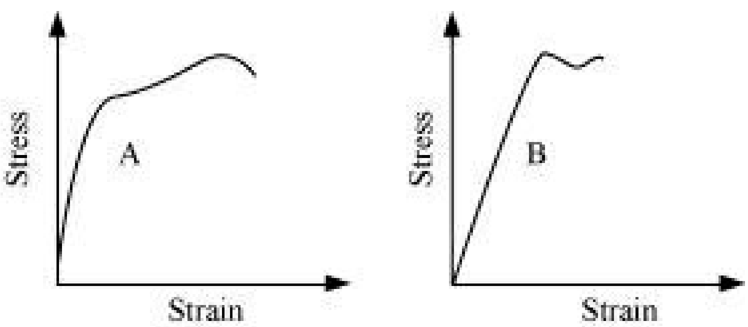
$$= \frac{150 \times 10^6}{0.002} = 7.5 \times 10^{10} N/m^2$$

Hence, Young's modulus for the given material is $7.5 \times 10^{10}\ N/m^2$.

(b) The yield strength of a material is the maximum stress that the material can sustain without crossing the elastic limit.

It is clear from the given graph that the approximate yield strength of this material is $300 \times 10^6\ Nm/^2\ or\ 3 \times 10^8\ N/m^2$.

24. The stress-strain graphs for materials A and B are shown in Fig. 9.12.



The graphs are drawn to the same scale.

(a) Which of the materials has the greater Young's modulus?

(b) Which of the two is the stronger material?

Ans. (a) A (b) A

(a) For a given strain, the stress for material **A** is more than it is for material **B**, as shown in the two graphs.

Young's modulus $= \frac{Stress}{Strain}$

For a given strain, if the stress for a material is more, then Young's modulus is also greater for that material. Therefore, Young's modulus for material **A** is greater than it is for material **B**.

(b) The amount of stress required for fracturing a material, corresponding to its fracture point, gives the strength of that material. Fracture point is the extreme point in a stress-strain curve. It can be observed that material **A** can withstand more strain than material **B**. Hence, material **A** is stronger than material **B**.

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25. Read the following two statements below carefully and state, with reasons, if it is true or false.

(a) The Young's modulus of rubber is greater than that of steel;

(b) The stretching of a coil is determined by its shear modulus.

Ans. (a) False **(b)** True

(a) For a given stress, the strain in rubber is more than it is in steel.

Young's modulus, Y $= \frac{Stress}{Strain}$

For a constant stress: $Y \propto = \frac{1}{Strain}$

Hence, Young's modulus for rubber is less than it is for steel.

(b) Shear modulus is the ratio of the applied stress to the change in the shape of a body. The stretching of a coil changes its shape. Hence, shear modulus of elasticity is involved in this process.

26. How much should the pressure on a litre of water be changed to compress it by 0.10%?

Ans. Volume of water, V = 1 L

It is given that water is to be compressed by 0.10%.

$\therefore$ Fractional change, $\frac{\Delta V}{V} = \frac{0.1}{100\times 1} = 10^{-3}$

Bulk modulus, $B = \frac{p}{\frac{\Delta V}{V}} \Rightarrow p = B\times\frac{\Delta V}{V}$

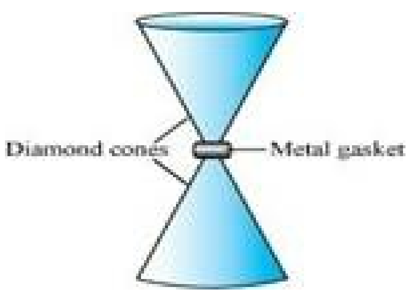
Bulk modulus of water, B= $2.2\times 10^{9}\,Nm^{-2}$

$p = 2.2\times 10^{9}\times 10^{-3}$

$= 2.2\times 10^{6}\,Nm^{-2}$

Therefore, the pressure on water should be $= 2.2\times 10^{6}\,Nm^{-2}$

27. Anvils made of single crystals of diamond, with the shape as shown in Fig. 9.14, are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?



Ans. Diameter of the cones at the narrow ends, d = 0.50 mm = 0.5×10^{-3} m

Radius, r = $\frac{d}{2}=0.25\times10^{-3}m$

Compression force, F = 50000 N

Pressure at the tip of the anvil: $p=\frac{Force}{Area\pi}=\frac{F}{r^2}$

$$\frac{50000}{\pi\left(0.25\times10^{-3}\right)^2}$$

$$=2.55\times10^{11}\,Pa$$

Therefore, the pressure at the tip of the anvil is 2.55 $\times10^{11}$ Pa.

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28. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one quarter of the load.

Ans. Diameter of the metal strip, d = 6.0 mm = 6.0×10^{-3} m

Radius, r = $\frac{d}{2}=3.0\times10^{-3}m$

Maximum shearing stress $=6.9\times10^7\;Pa$

Maximum stress = $\frac{Maximum\ load\ or\ force}{Area}$

Maximum force = Maximum stress $\times$ Area

= $6.9\times10^7\times\pi\times(r)^2$

= $6.9\times10^7\times\pi\times\left(3\times10^{-3}\right)^2$

= 1949.94 N

Each rivet carries one quarter of the load.

∴ Maximum tension on each rivet = 4 $\times$ 1949.94 = 7799.76 N

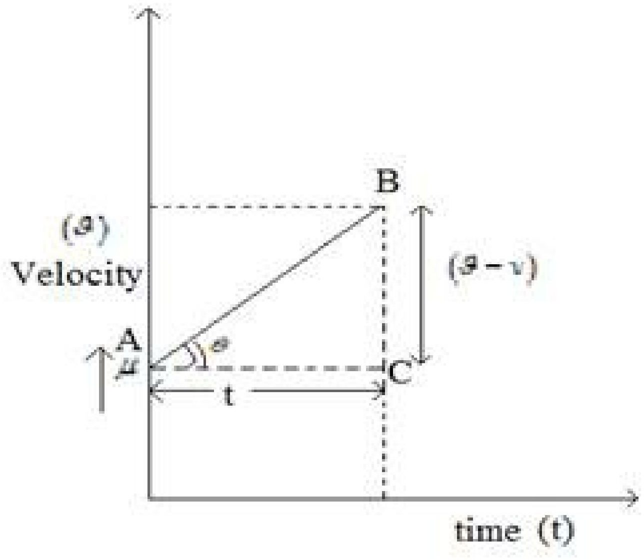
3 Marks Questions

1.Define $\vartheta = v + at$ from velocity time graph.

Ans: Slope of $\upsilon - t$ graph

$$\tan\theta = \frac{\upsilon - v}{t}$$

$$But \tan\theta = \text{acceleration(a)}$$



$$\Rightarrow a = \frac{\upsilon - v}{t}$$

$$\upsilon - v = at$$

$$\upsilon = v + at$$

2.A particle is moving along a straight line and its position is given by the relation

$$x = \left(t^3 - 6t^2 - 15t + 40\right)m$$

Find (a) The time at which velocity is zero.

(b) Position and displacement of the particle at that point.

(c) Acceleration

Ans: $x = t^3 - 6t^2 - 15t + 40$

$$v = \frac{dx}{dt} = (3t^2 - 12t - 15)\, m/s$$

$$a = \frac{dv}{dt} = (6t - 12)\, m/s^2$$

(a) $3t^2 - 12t - 15 = 0$

$3t^2 - 15t + 3t - 15 = 0$

$3t(t-5) + 3(t-5) = 0$

$(3t+3)(t-5) = 0$

Either $t = -1$ or $t = 5$

Time cannot be negative

∴ t = 5 seconds.

(b) Position at t = 5 s   At t = 0 s

$x = (5)^3 - 6(5)^2 - 15(5) + 40$
$x = 40m$

$x = -60m$

Displacement at t = 5 s and t = 0s

$s = x_5 - x_0$

$x_5 = -60m$

$x_0 = 40m$

$s = -60 - 40$

$s = -100m$

c) Acceleration at t = 5s

$a = 6(5) - 12$

$a = (30 - 12)$

$a = 18 m/s^2$

leration for the particle at that line.

3.A police jeep on a petrol duty on national highway was moving with a speed of 54km/hr. in the same direction. It finds a thief rushing up in a car at a rate of 126km/hr in the same direction. Police sub – inspector fired at the car of the thief with his service revolver with a muzzle speed of 100m/s. with what speed will the bullet hit the car of thief?

Ans: V_{PJ} = 54km/hr = 15m/s V_{TC} = 126km/hr = 35m/s

Muzzle speed of the bullet $v_b = 100m/s$.

V_{CP} = 35 – 15 = 20m/s. V_{CP} = Velocity of car w.r.t. police

V_{BC}= 100 – 20 = 80 m/s V_{BC} = Velocity of bullet w.r.t car

Thus bullet will hit the car with a velocity 80m/s.

4.Establish the relation $Snth = u + a/2(2n-1)$ **where the letters have their usual meanings.**

Ans: $Snth = Sn - Sn_{-1}$

$$S_n = un + \frac{1}{2}an^2$$

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$Snth = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

$$\cancel{un} + \frac{\cancel{1}}{\cancel{2}}\cancel{an}^2 - \cancel{un} + u - \frac{\cancel{1}}{\cancel{2}}\cancel{an}^2 - \frac{1}{2}a + na$$

$$\text{Snth} = \text{u-}\frac{1}{2}a + na$$

$$= \text{u+}\frac{a}{2}(2n-1)$$

Hence proved.

5.A stone is dropped from the top of a cliff and is found to ravel 44.1m diving the last second before it reaches the ground. What is the height of the cliff? g = 9.8m/s^2

Ans: Let h be the height of the cliff

n be the total time taken by the stone while falling

u = 0

A = g = 9.8m/s^2

$$Snth = u + \frac{a}{2}(2n-1)$$

$$44.1 = 0 + \frac{9.8}{2}(2n-1)$$

$n = {}^{10}\!/_{2} = 55$

Height of the cliff

$$h = ut + \frac{1}{2}at^2$$

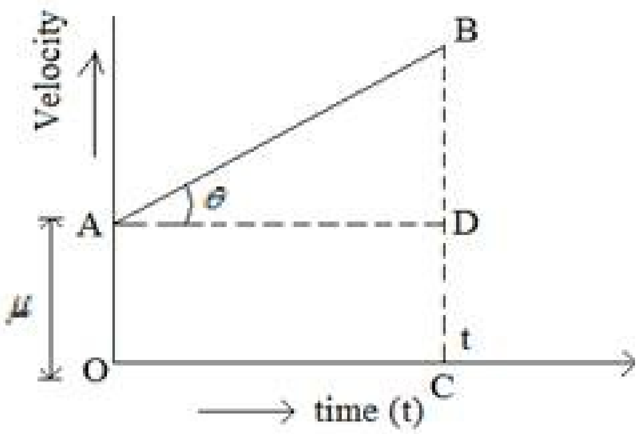
$$h = un + \frac{1}{2}gn^2$$

$$h = 0 \times 5 + \frac{1}{2} \times 9.8 \times (5)^2$$

$$h = 4.9 \times 25$$

h = 122.5m

6.Establish $s = ut + \frac{1}{2}at^2$ from velocity time graph for a uniform accelerated motion?



Ans: Displacement of the particle in time (t)

S = area under $\upsilon - t$ graph

S = area OABC

S = area of rectangle AODC + area of Δ ADB

$$S = OA \times OC + \frac{1}{2} AD \times BD$$

$$S = ut + \frac{1}{2}(AD) \times \left(\frac{AD \times DB}{AD}\right)$$

$$S = ut + \frac{1}{2}(AD)^2 \times \left(\frac{DB}{AD}\right)$$

$$S = ut + \frac{1}{2}(t)^2 \times \left(\frac{DB}{AD}\right)$$

$$S = ut + \frac{1}{2}(t)^2 \times (a)$$

$$\left[\because a = \tan\theta = \frac{BD}{AD}\right]$$

$$S = ut + \frac{1}{2}at^2$$

7.(a) Define the term relative velocity?

(b) Write the expression for relative velocity of one moving with respect to another body when objects are moving in same direction and are moving in opposite directions?

(c) A Jet airplane traveling at the speed of 500km/hr ejects its products of combustion at the speed of 1500km/h relative to the Jet plane. What is the speed of the latter with respect to an observer on the ground?

Ans: (a) Relative velocity $\overline{V}_{AB}$ of body A with respect to body B is defined as the time rate of change of position of A wrt. B.

(b) (i) When two objects move in the same direction

$$\overrightarrow{VAB} = \overrightarrow{VA} - \overrightarrow{VB} \quad A \rightarrow \overrightarrow{VA}$$

$$B \rightarrow \overrightarrow{VB}$$

$$\rightarrow \overrightarrow{VAB}$$

(ii) When two objects move in the opposite direction

$$\overrightarrow{VAB} = \overrightarrow{VA} - (-\overrightarrow{VB}) \quad A \rightarrow \overrightarrow{v_A}$$

$$\overrightarrow{VAB} = \overrightarrow{VA} + \overrightarrow{VB} \quad \leftarrow \overrightarrow{vB}$$

$$\rightarrow \overrightarrow{VAB}$$

(c) Velocity of the Jet plane V_J = 500km/hr velocity of gases wrt. Jet plane V_{gJ} = -1500km/hr (direction is opposite)

$$V_{gJ} = V_g - V_J$$

$$V_g = V_{gJ} + V_J$$

Velocity of the Vg = -1500 + 500 = -1000km/hr

(As hot gases also comes out in opposite direction of the Jet plane)

8.Define (i) v = u + at (ii) $V^2 - u^2 = 2as$ by calculus method

Ans:We know

(i) $a = \frac{dv}{dt}$

$$dv = adt$$

Integrating $\int dv = \int adt$

$$V = at + k - - - -(1)$$

Where K is constant of integration

$when\ t = 0\ \vartheta = u$

$\Rightarrow K = u$

$\Rightarrow V = at + u$

(ii) $\upsilon^2 - v^2 = 2as$

We know $a = \frac{dv}{dt}$

Multiply and Divide by dx

$$a = \frac{dv}{dt} \times \frac{dx}{dx}$$

$$a = \frac{dv}{dx} \times \vartheta$$

$$adx = vdv$$

$$\left(\because \frac{dx}{dt} = v \right)$$

Integrating within the limits

$$a \int_{x_o}^{x} dx = \int_{v}^{\vartheta} \upsilon dv$$

$$a(x - x_o) = \frac{\upsilon^2}{2} - \frac{v^2}{2}$$

$$as = \frac{\upsilon^2 - v^2}{2}$$

$$\left(\because (x - x_o) = s = \text{displacement} \right)$$

$$\upsilon^2 - v^2 = 2as$$

9.Explain :-

1) Elastic Body   2) Plastic Body   3) Elasticity.

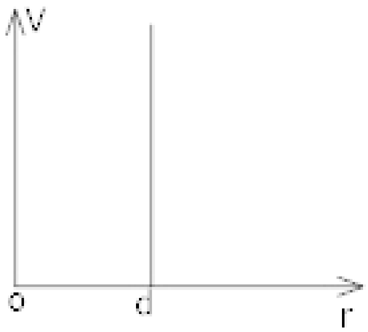
Ans.1) Elastic Body → A body which completely regains its original configuration immediately after the removal of deforming force on it is called elastic body. eg. Quartz and phosphor Bronze.

2) Plastic Body → A body which does not regain its original configuration at all on the removal of deforming force, howsoever the deforming force may be is called plastic body eg:- Paraffin wax.

3) Elasticity → The property of the body to regain its original configuration, when the deforming forces are removed is called plasticity.

l= Original Length of wire.

10.Why is the force of repulsion responsible for the formation of a solid and not the forces of attraction?



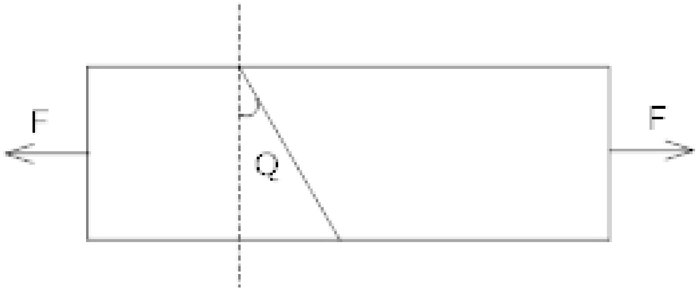
Ans. If we study the motion of large number of spheres, it will be observed that two hard spheres do not attract each other, but rebound immediately on collision. That is they, do not come closer than their diameter ‘d’. The interaction potential ‘V’ for a pair of hard sphere is

d = diameter

r = distance of interaction of 2 spheres

It shows that there is infinite repulsion for r = d and no potential for r>d and hence repulsive forces binds them together.

11.A bar of cross section A is subjected to equal and opposite tensile force F at its ends. If there is a plane through the bar making an angle Q, with the plane at right angles to the bar in the figure



a) Find the tensile stress at this plane in terms of F, A and Q

b) What is the shearing stress at the plane in terms of F, and Q.

c) For what value of Q is tensile stress a maximum.

Ans. 1) Tensile stress = $\frac{Normal\ force}{Area}$

Normal force = F cos θ

Tensile Stress = $\frac{\frac{F\cos\theta}{A}}{\cos\theta} = \frac{F\cos^2\theta}{A}$

2) Shearing Stress = $\frac{Tangential\ Stress}{Area}$

Tangential = F Sin θ

force

Area = A/cos θ

Shearing Stress = $\frac{\frac{F\sin\theta}{A}}{\cos\theta}$

$= \frac{F}{A} Sin\theta \ \cos\theta$

$= \frac{F 2 \sin\theta \cos\theta}{2A}$ (Divide & Multiply by 2)

$= \frac{F\ Sin^2\theta}{2A} \left(\because\ Sin^2\theta = 2Sin\theta \cos\theta\right)$

3) Tensile Stress = $\frac{F \cos^2\theta}{A}$, for A & F constant

Tensile stress α cos2 θ

cos 2 θ = Maximum = 1

cos θ = 1

θ = $\cos^{-1}(1)$

$\theta = 0^0$

i.e. when the plane is parallel to the bar.

c) For what value of Q is tensile stress a maximum.

12.The Young's modulus of steel is $2.0\text{x}10^{+11}$ w/m^2. If the interatomic spacing for the metal is $2.8\text{x}10^{-10}$m, find the increase in the interatomic spacing for a force of 10^9 N|m^2 and the force constant?

Ans.Y = $2.0\text{x}10^{11}$N|m^2

L = $2.8\text{x}10^{-10}$m

F = force

A = Area

Δl = charge in length

$\frac{F}{A} = 10^9 N|m^2$; force constant = K = $\frac{F}{\Delta l}$

Δl = ? ; $\frac{F}{\Delta l} = ?$

So, Y = Modulus of elasticity = $\frac{F \times \ell}{A \times \Delta \ell}$

Or Δl = $\frac{F}{A} X \frac{l}{y}$

Δl = $\frac{10^9 x 2.8 x 10^{-10}}{2 x 10^{11}}$

$$\Delta l = \frac{2.8 x 10^{-10+9-11}}{2}$$

$$\Delta l = \frac{2.8 x 10^{-12}}{2}$$

$\Delta l = 1.4 x 10^{-12} m$ (lA^0 = 10^{-10}m)

$\Delta \ell = 0.014 A^0$

As the distance between 2 atoms is l then area of chain of atoms = A = l x l = l^2 → (1)

Y = $\frac{F \times \ell}{A \times \Delta \ell} = \frac{F \times \ell}{\Delta \ell \times A}$

Y = $\frac{F x \ell}{\Delta \ell \times \ell^2}$ ($\because$ Using equation i)

Y = $\frac{F x \ell}{\Delta \ell \times \ell}$

(Force constant) = K = $\frac{F}{\Delta \ell}$; so

Y = $\frac{K}{l}$

K = Yl P

K = $2.0 \times 10^{11} \times 2.8 \times 10^{-10}$

K = $2 \times 2.8 \times 10^{11-10}$

K = 5.6×10^{4}

K = $56 Nm^{-1}$

---

13. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.

Ans. Mass of the big structure, *M* = 50,000 kg

Inner radius of the column, *r* = 30 cm = 0.3 m

Outer radius of the column, *R* = 60 cm = 0.6 m

Young's modulus of steel, *Y* = $2 \times 10^{11}\ Pa$

Total force exerted, *F* = *M*g = 50000 $\times$ 9.8 N

Stress = Force exerted on a single column $= \frac{50000 \times 9.8}{4}$ = 122500 N

Young's modulus, $Y = \frac{Stress}{Strain}$

$$Strain = \frac{\frac{F}{A}}{Y}$$

Where,

Area, $A = \pi\left(R^2 - r^2\right) = \pi\left((0.6)^2 - (0.3)^2\right)$

$$Strain = \frac{122500}{\pi\left[(60)^2 - (0.30)^2\right] \times 2 \times 10^{11}}$$

$= 7.22 \times 10^{-7}$

Hence, the compressional strain of each column is 7.22×10^{-7}.

---

14. A piece of copper having a rectangular cross-section of 15.2 mm$\times$19.1 mm is pulled intension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain?

Ans. Length of the piece of copper, l = 19.1 mm = $19.1 \times 10^{-3}\ m$

Breadth of the piece of copper, b = 15.2 mm = $15.2 \times 10^{-3}\ m$

Area of the copper piece:

A = $l \times b$

= $19.1 \times 10^{-3} \times 15.2 \times 10^{-3}$

= $2.9 \times 10^{-4}\ m^2$

Tension force applied on the piece of copper, F = 44500 N

Modulus of elasticity of copper, $\eta = 42 \times 10^9\ N/m^2$

Modulus of elasticity, $\eta = \frac{Stress}{Strain} = \frac{\frac{F}{A}}{Strain}$

$$\therefore Strain = \frac{F}{A\eta}$$

$$= \frac{44500}{2.9\times10^{-4}\times42\times10^{9}}$$

$= 3.65\times10^{-3}$

---

15. The edge of an aluminum cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

Ans. Edge of the aluminium cube, L = 10 cm = 0.1 m

The mass attached to the cube, m = 100 kg

Shear modulus (η) of aluminium = $25\ GPa = 25\times10^{9}\ Pa$

Shear modulus,

$$\eta = \frac{Shear\ stress}{Shear\ strain} = \frac{\frac{F}{A}}{\frac{L}{\Delta L}}$$

Where,

F = Applied force = mg = 100 × 9.8 = 980 N

A = Area of one of the faces of the cube = 0.1 × 0.1 = 0.01 m^2

ΔL = Vertical deflection of the cube

$$\therefore \Delta L = \frac{FL}{A\eta}$$

$$= \frac{980\times0.1}{10^{-2}\times\left(25\times10^{9}\right)}$$

$= 3.92\times10^{-7}\ m$

The vertical deflection of this face of the cube is $3.92\times10^{-7}\ m$.

4 Marks Questions

1. Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013×10^5 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Ans. Initial volume, $V_1 = 100.0 l = 100.0\times10^{-3}\ m^3$

Final volume, $V_2 = 100.5\ l = 100.5\times10^{-3}\ m^3$

Increase in volume, $\Delta V = V_2 - V_1 = 0.5\times10^{-3}\ m^3$

Increase in pressure, Δp = 100.0 atm = 100 × 1.013 × 105 Pa

Bulk modulus = $\frac{\Delta p}{\frac{\Delta V}{V_1}} = \frac{\Delta p\times V_1}{\Delta V}$

$= \frac{100\times1.013\times10^5\times100\times10^{-3}}{0.5\times10^{-3}}$

$= 2.026\times10^9\ Pa$

Bulk modulus of air $= 1.0\times10^5\ Pa$

$\therefore \frac{Bulk\ modulus\ of\ water}{Bulk\ modulus\ of\ air} = \frac{2.026\times10^9}{1.0\times10^5} = 2.026\times10^4$

This ratio is very high because air is more compressible than water.

---

2. The Marina trench is located in the Pacific Ocean, and at one place it is nearly eleven km beneath the surface of water. The water pressure at the bottom of the trench is about 1.1×10^8 Pa. A steel ball of initial volume 0.32 m^3 bis dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches to the bottom?

Ans. Water pressure at the bottom, $p = 1.1 \times 10^8$ **Pa**

Initial volume of the steel ball, V = 0.32 m^3

Bulk modulus of steel, $B = 1.6 \times 10^{11}\ Nm^{-2}$

The ball falls at the bottom of the Pacific Ocean, which is 11 km beneath the surface.

Let the change in the volume of the ball on reaching the bottom of the trench be ΔV.

Bulk modulus, B = $\frac{p}{\frac{\Delta V}{V}}$

$$\Rightarrow \Delta V = \frac{B}{pV}$$

$$= \frac{1.1 \times 10^8 \times 0.32}{1.6 \times 10^{11}}$$

$$= 2.2 \times 10^{-4} m^3$$

Therefore, the change in volume of the ball on reaching the bottom of the trench is $2.2 \times 10^{-4}\ m^3$.

---

3. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratio of their diameters if each is to have the same tension.

Ans. The tension force acting on each wire is the same. Thus, the extension in each case is the same. Since the wires are of the same length, the strain will also be the same.

The relation for Young's modulus is given as:

$$Y = \frac{Stress}{Strain} = \frac{\frac{F}{A}}{Strain} = \frac{\frac{4F}{\pi d^2}}{Strain} \quad ....(i)$$

Where,

F = Tension force

A = Area of cross-section

d = Diameter of the wire

It can be inferred from equation (*i*) that $Y \propto \frac{1}{d^2}$

Young's modulus for iron, $Y_1 = 190 \times 10^9\ Pa$

Diameter of the iron wire = d_1

Young's modulus for copper, $Y_2 = 190 \times 10^9\ Pa$

Diameter of the copper wire = d_2

Therefore, the ratio of their diameters is given as:

$$\frac{d_2}{d_1} = \sqrt{\frac{Y_1}{Y_2}} = \sqrt{\frac{190 \times 10^9}{120 \times 10^9}} = \sqrt{\frac{19}{12}} = 1.25 : 1$$

5 Marks Questions

1. A steel wire of length 4.7 m and cross-sectional area $3.0\times10^{-5}\ m^2$ stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of $4.0\times10^{-5}\ m^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Ans. Length of the steel wire,

$$\therefore \Delta l = 2\left[(0.5)^2 + l^2\right]^{\frac{1}{2}} - 1.0$$

L_1 = 4.7 m

Area of cross-section of the steel wire, $A_1 = 3.0\times10^{-5}\ m^2$

Length of the copper wire, L_2 = 3.5 m

Area of cross-section of the copper wire, $A_2 = 4.0\times10^{-5}\ m^2$

Change in length = ΔL_1 = ΔL_2 = ΔL

Force applied in both the cases = F

Young's modulus of the steel wire:

$$Y_1 = \frac{F_1}{A_1}\times\frac{l_1}{\Delta L_1}$$

$$= \frac{F\times4.7}{3.0\times10^{-5}\times\Delta L} \quad \ldots (i)$$

Young's modulus of the copper wire:

$$Y_2 = \frac{F_2}{A_2}\times\frac{l_2}{\Delta L_2}$$

$$= \frac{F\times3.5}{4.0\times10^{-5}\times\Delta L} \quad \ldots (ii)$$

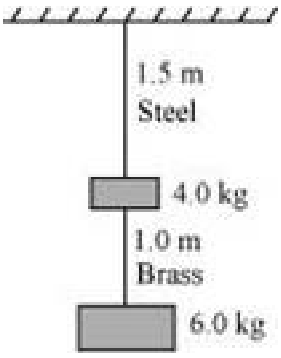
Dividing (*i*) by (*ii*), we get:

$$\frac{Y_1}{Y_2} = \frac{4.7 \times 4.0 \times 10^{-5}}{3.0 \times 10^{-5} \times 3.5} = 1.79 : 1$$

The ratio of Young's modulus of steel to that of copper is 1.79: 1.

---

2. Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Fig. 9.13. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.



Ans. Elongation of the steel wire = 1.49×10^{-4} m

Elongation of the brass wire = 1.3×10^{-4} m

Diameter of the wires, d = 0.25 m

Hence, the radius of the wires, $r = \frac{d}{2}$ = 0.125 cm

Length of the steel wire, L_1 = 1.5 m

Length of the brass wire, L_2 = 1.0 m

Total force exerted on the steel wire:

$$F_1 = (4 + 6)\ g = 10\times9.8 = 98\ N$$

Young's modulus for steel:

$$Y_1 = \frac{\left(\frac{F_1}{A_1}\right)}{\left(\frac{\Delta L_1}{L_1}\right)}$$

Where,

ΔL_1 = Change in the length of the steel wire

A_1 = Area of cross-section of the steel wire $= \pi r_1^2$

Young's modulus of steel, $Y_1 = 2.0\times10^{11}\ Pa$

$$\therefore \Delta L_1 = \frac{F_1\times L_1}{A_1\times Y_1} = \frac{F_1\times L_1}{\pi r_1^2\times Y_1}$$

$$= \frac{98\times1.5}{\pi\left(0.125\times10^{-2}\right)^2\times2\times10^{11}} = 1.49\times10^{-4}\ m$$

Total force on the brass wire:

$$F_2 = 6\times9.8 = 58.8\ N$$

Young's modulus for brass:

$$Y_2 = \frac{\left(\frac{F_2}{A_2}\right)}{\left(\frac{\Delta L_2}{L_2}\right)}$$

Where,

ΔL_2 = Change in length

A_2 = Area of cross-section of the brass wire

$$\therefore \Delta L_2 = \frac{F_2 \times L_2}{A_2 \times Y_2} = \frac{F_2 \times L_2}{\pi r_2^2 \times Y_2}$$

$$= \frac{58.8 \times 1.0}{\pi\left(0.125 \times 10^{-2}\right)^2 \times \left(0.91 \times 10^{11}\right)} = 1.3 \times 10^{-4} m$$

Elongation of the steel wire = $1.49 \times 10^{-4}\ m$

Elongation of the brass wire = $1.3 \times 10^{-4}\ m$

---

3. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2 . Calculate the elongation of the wire when the mass is at the lowest point of its path.

Ans. Mass, *m* = 14.5 kg

Length of the steel wire, *l* = 1.0 m

Angular velocity, ω = 2 rev/s = 2 $\times$ 2π rad/s = 12.56 rad/s

Cross-sectional area of the wire, $a = 0.065\ cm^2 = 0.065 \times 10^{-4}\ m^2$

Let Δ*l* be the elongation of the wire when the mass is at the lowest point of its path.

When the mass is placed at the position of the vertical circle, the total force on the mass is:

F = *mg* + ml^2

= $14.5 \times 9.8 + 14.5 \times 1 \times (12.56)^2$

= 2429.53 N

Young's modulus $= \frac{Stress}{Strain}$

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}} = \frac{F}{A}\frac{l}{\Delta l}$$

$$\therefore \Delta l = \frac{Fl}{AY}$$

Young's modulus for steel = $2\times10^{11}\ Pa$

$$\Delta l = \frac{2429.53\times1}{0.065\times10^{4}\times2\times10^{11}}$$

$$\Rightarrow \Delta l = 1.87\times10^{-3} m$$

Hence, the elongation of the wire is $1.87\times10^{-3}\ m$.

---

4. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×10^{3} kg m^{-3}?

Ans. Let the given depth be *h*.

Pressure at the given depth, *p* = 80.0 atm = $80\times1.01\times10^{5}\ Pa$

Density of water at the surface, $p_1 = 1.03\times10^{3}\ kg\ m^{-3}$

Let ρ_2 be the density of water at the depth *h*.

Let V_1 be the volume of water of mass *m* at the surface.

Let V_2 be the volume of water of mass *m* at the depth *h*.

Let ΔV be the change in volume.

$$\Delta V = V_1 - V_2$$

$$= m\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$$

$\therefore$ Volumetric strain = $\frac{\Delta V}{V_1}$

$$= m\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) \times \frac{\rho_1}{m}$$

$$\therefore \frac{\Delta V}{V_1} = 1 - \frac{\rho_1}{\rho_2} \text{ .... (i)}$$

Bulk modulus, $B = \frac{\Delta V}{V_1}$

$$\frac{\Delta V}{V_1} = \frac{\rho}{B}$$

Compressibility of water $= \frac{1}{B} = 45.8 \times 10^{-11} Pa^{-1}$

$$\therefore \frac{\Delta V}{V_1} = 80 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 3.71 \times 10^{-3} \text{ .... (ii)}$$

For equations (*i*) and (*ii*), we get:

$$1 - \frac{\rho_1}{\rho_2} = 3.71 \times 10^{-3}$$

$$\rho_2 = \frac{1.03 \times 10^3}{1 - \left(3.71 \times 10^{-3}\right)}$$

$$= 1.034 \times 10^3 \, kg\, m^{-3}$$

Therefore, the density of water at the given depth (*h*) is $= 1.034 \times 10^3 \, kg\, m^{-3}$.

5. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.

Ans. Length of an edge of the solid copper cube, *l* = 10 cm = 0.1 m

Hydraulic pressure, *p* = 7.0×10^6 Pa

Bulk modulus of copper, *B* = 140×10^9 Pa

Bulk modulus, $B = \frac{p}{\frac{\Delta V}{V}}$

Where, $\frac{\Delta V}{V}$ = Volumetric strain

Δ*V* = Change in volume

V = Original volume.

$$V = \frac{pV}{B}$$

Original volume of the cube, *V* = l^3

$$\therefore \Delta V = \frac{pl^3}{B}$$

$$= \frac{7 \times 10^6 \times (0.1)^3}{140 \times 10^9}$$

$$= 5 \times 10^{-8} m^3$$

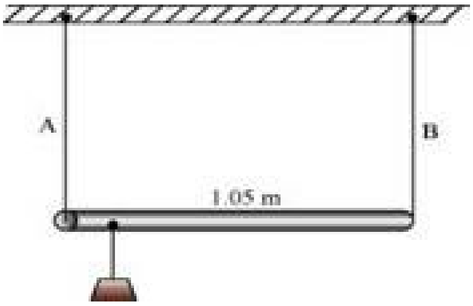
$$= 5 \times 10^{-2} cm^{-3}$$

Therefore, the volume contraction of the solid copper cube is $5 \times 10^{-2} cm^{-3}$.

---

6. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of

steel (wire A) and aluminium (wire B) of equal lengths as shown in Fig. 9.15. The cross-sectional areas of wires A and B are 1.0 mm2 and 2.0 mm2, respectively. At what point along the rod should a mass *m* be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.



Ans. (a) 0.7 m from the steel-wire end

(b) 0.432 m from the steel-wire end

Cross-sectional area of wire **A**, $a_1 = 1.0\ mm^2 = 1.0\times10^{-6}\ m^2$

Cross-sectional area of wire **B**, $a_2 = 2.0\ mm^2 = 2.0\times10^{-6}\ m^2$

Young's modulus for steel, $Y_1 = 2\times10^{11}\ Nm^{-2}$

Young's modulus for aluminium, $Y_2 = 7.0\times10^{10}\ Nm^{-2}$

(a) Let a small mass *m* be suspended to the rod at a distance *y* from the end where wire **A** is attached.

Stress in the wire = $\dfrac{Force}{Area} = \dfrac{F}{a}$

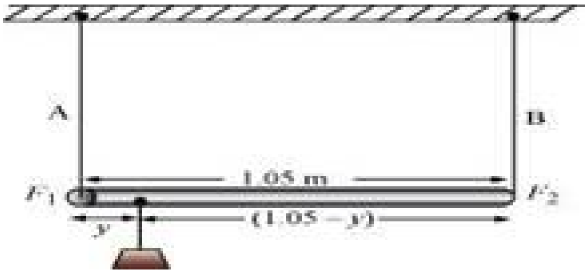
If the two wires have equal stresses, then: $\dfrac{F_1}{a_1} = \dfrac{F_2}{a_2}$

Where, F_1 = Force exerted on the steel wire

F_2 = Force exerted on the aluminum wire

$$\frac{F_1}{F_1}=\frac{a_2}{a_2}=\frac{1}{2}$$ ....(i)

The situation is shown in the following figure.



Taking torque about the point of suspension, we have:

$$F_1 y = F_2(1.05-y)$$

$$\frac{F_1}{F_2}=\frac{(1.05-y)}{1}$$ ..... (ii)

Using equations (*i*) and (*ii*), we can write:

$$\frac{(1.05-y)}{y}=\frac{1}{2}$$

$$2(1.05-y)=y$$

$$2.1-2y=y$$

$$3y=2.1$$

$$\therefore y=0.7m$$

In order to produce an equal stress in the two wires, the mass should be suspended at a distance of 0.7 m from the end where wire **A** is attached.

(b)Young's modulus = $\frac{Stress}{Strain}$

$$Strain = \frac{Stress}{Young's\ modulus} = \frac{\frac{F}{a}}{Y}$$

If the strain in the two wires is equal, then:

$$\frac{\frac{F_1}{a_1}}{Y_1} = \frac{\frac{F_2}{a_2}}{Y_2}$$

$$\frac{F_1}{F_2} = \frac{a_1}{a_2}\frac{Y_1}{Y_2} = \frac{1}{2} \times \frac{2\times10^{11}}{7\times10^{10}} = \frac{10}{7} \text{ .... (iii)}$$

Taking torque about the point where mass *m*, is suspended at a distance *y*1 from the side where wire **A** attached, we get:

$$F_1 y_1 = F_2 (1.05 \ - y_1)$$

$$\frac{F_1}{F_2} = \frac{(1.05 - y)}{1} \text{ .... (iii)}$$

Using equations (*iii*) and (iv), we get:

$$\frac{(1.05 - y_1)}{y_1} = \frac{10}{7}$$

$$7(1.05 - y_1) = 10y_1$$

$$17y_1 = 7.35$$

$$\therefore y_1 = 0.432m$$

In order to produce an equal strain in the two wires, the mass should be suspended at a distance of 0.432 m from the end where wire **A** is attached.

---

7. A mild steel wire of length 1.0 m and cross-sectional area 0.50×10^{-2} cm^2 is

stretched, well within its elastic limit, horizontally between two pillars. A mass of 100 g is suspended from the mid-point of the wire. Calculate the depression at the midpoint.

Ans. 

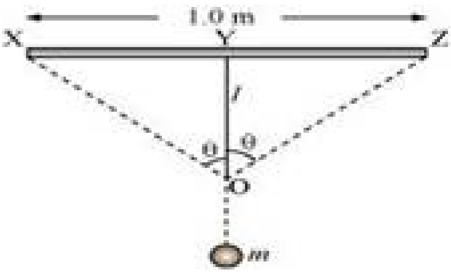
Length of the steel wire = 1.0 m

Area of cross-section, A = $0.50\times10^{-2}\,cm^2 = 0.50\times10^{-6}\ m^2$

A mass 100 g is suspended from its midpoint.

m = 100 g = 0.1 kg

Hence, the wire dips, as shown in the given figure.



Original length = XZ

Depression = l

The length after mass m, is attached to the wire = XO + OZ

Increase in the length of the wire:

Δl = (XO + OZ) – XZ

Where, XO = OZ = $\left[(0.5)^2 + l^2\right]^{\frac{1}{2}}$

$$\therefore \Delta = 2\left[(0.5)^2 + l^2\right]^{\frac{1}{2}} - 1.0$$

$$= 2\times0.5\left[1+\left(\frac{1}{0.5}\right)^2\right]^{\frac{1}{2}} - 1.0$$

Expanding an neglecting higher terms, we get:

$$\Delta l = \frac{l^2}{0.5}$$

Strain = $\frac{Increase\ in\ length}{Original\ length}$

Let T be the tension in the wire.

$\therefore mg = 2T\cos\theta$

Using the figure, it can be written as:

$$\cos\theta = \frac{l}{\left((0.5)^2 + l^2\right)^{\frac{1}{2}}}$$

$$= \frac{l}{(0.5)\left(1 + \left(\frac{l}{0.5}\right)^2\right)^{\frac{1}{2}}}$$

Expanding the expression and eliminating the higher terms:

$$\cos\theta = \frac{l}{(0.5)\left(1 + \frac{l^2}{2(0.5)^2}\right)}$$

$$\left(- + \frac{l^2}{0.5}\right) \doteq 1 \ for\ small\ l$$

$$\therefore \cos\theta = \frac{l}{0.5}$$

$$\therefore T = \frac{mg}{2\left(\frac{l}{0.5}\right)} = \frac{mg \times 0.5}{2l} = \frac{mg}{4l}$$

Stress = $\frac{Tension}{Area} = \frac{mg}{4l \times A}$

Young’s modulus = $\frac{Stress}{Strain}$

$$l = \sqrt{\frac{mg \times 0.5}{4YA}}$$

Young's modulus of steel, Y = $2 \times 10^{11}\ Pa$

$$\therefore l = \sqrt{\frac{0.1 \times 9.8 \times 0.5}{4 \times 2 \times 10^{11} \times 0.50 \times 10^{-6}}}$$

$$= 0.0106 m$$

Hence, the depression at the midpoint is 0.0106 m.